Solve Rational Equations and Inequalities

Solve Rational Equations Algebraically

Solve.

a) $\dfrac{4}{3x - 5} = 4$

b) $\dfrac{x - 5}{x^2 - 3x - 4} = \dfrac{3x + 2}{x^2 - 1}$

Example 2 Solve a Rational Equation Using Technology

Solve $\dfrac{x}{x - 2} = \dfrac{2x^2 - 3x + 5}{x^2 + 6}$.

Example 3 Solve a Simple Rational Inequality

Solve $\dfrac{2}{x-5} < 10$.

Example 4 Solve a Quadratic Over a Quadratic Rational Inequality

Solve $\dfrac{x^2 - x - 2}{x^2 + x - 12} \geq 0$.

KEY CONCEPTS

- To solve rational equations algebraically, start by factoring the expressions in the numerator and denominator to find asymptotes and restrictions.
- Next, multiply both sides by the factored denominators, and simplify to obtain a polynomial equation. Then, solve using techniques from Chapter 2.
- For rational inequalities:
  - It can often help to rewrite with the right side equal to 0. Then, use test points to determine the sign of the expression in each interval.
  - If there is a restriction on the variable, you may have to consider more than one case. For example, if $\frac{a}{x-k} < b$, case 1 is $x > k$ and case 2 is $x < k$.
- Rational equations and inequalities can be solved by studying the key features of a graph with paper and pencil or with the use of technology.
- Tables and number lines can help organize intervals and provide a visual clue to solutions.

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Intervals of Increase and Decrease

We say that a function f *is decreasing on an interval* if, for any value of $x_1 < x_2$ in the interval, $f(x_1) > f(x_2)$.

Similarly, we say that a function f *is increasing on an interval* if, for any value of $x_1 < x_2$ in the interval, $f(x_1) < f(x_2)$.