

# Ex2.6 (p. 138 - 139) # 1 - 13

## Case 2

Both factors are non-positive, and  $w$  is positive (because  $w$  represents the width).

$$0 < w \leq 8 \quad w^2 + 19w + 176 \leq 0$$

$w^2 + 19w + 176 \leq 0$  is not possible for any values of  $w$ . There is no solution.

So, the possible solution is  $w \geq 8$ .

When  $w = 8$ ,  $h = \frac{1}{4}(8) + 2 = 4$  and  $l = 4(2) + 12 = 20$ .

The dimensions of the excavation that give a volume of at least  $1408 \text{ cm}^3$  are width 8 m, depth 4 m, and length 20 m.

## KEY CONCEPTS

- Factorable inequalities can be solved algebraically by
  - considering all cases
  - using intervals and then testing values in each interval
- Tables and number lines can help organize intervals to provide a visual clue to solutions.

## Communicate Your Understanding

- C1 Why is it necessary to reverse an inequality sign when each side is multiplied or divided by a negative value? Support your answer with examples.
- C2 What are the similarities between solving a linear inequality and solving a polynomial inequality?
- C3 Which method is more efficient for solving factorable inequalities algebraically, using cases or using intervals? Explain.

## A Practise

For help with question 1, refer to Example 1.

1. Solve each inequality. Show each solution on a number line.
  - a)  $x + 3 \leq 5$
  - b)  $2x + 1 > -4$
  - c)  $5 - 3x \geq 6$
  - d)  $7x < 4 + 3x$
  - e)  $2 - 4x > 5x + 20$
  - f)  $2(1 - x) \leq x - 8$

For help with questions 2 to 4, refer to Example 2.

2. Solve by considering all cases. Show each solution on a number line.
  - a)  $(x + 2)(x - 4) > 0$
  - b)  $(2x + 3)(4 - x) \leq 0$

3. Solve using intervals. Show each solution on a number line.

- a)  $(x + 3)(x - 2) > 0$
- b)  $(x - 6)(x - 9) \leq 0$
- c)  $(4x + 1)(2 - x) \geq 0$

4. Solve.

- a)  $(x + 2)(3 - x)(x + 1) < 0$
- b)  $(-x + 1)(3x - 1)(x + 7) \geq 0$
- c)  $(7x + 2)(1 - x)(2x + 5) > 0$
- d)  $(x + 4)(-3x + 1)(x + 2) \leq 0$

## B Connect and Apply

5. Solve by considering all cases. Show each solution on a number line.

- $x^2 - 8x + 15 \geq 0$
- $x^2 - 2x - 15 < 0$
- $15x^2 - 14x - 8 \leq 0$
- $x^3 - 2x^2 - 5x + 6 < 0$
- $2x^3 + 3x^2 - 2x - 3 \geq 0$

6. Solve using intervals.

- $x^3 + 6x^2 + 7x + 12 \geq 0$
- $x^3 + 9x^2 + 26x + 24 < 0$
- $5x^3 - 12x^2 - 11x + 6 \leq 0$
- $6x^4 - 7x^3 - 4x^2 + 8x + 12 > 0$

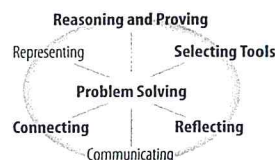
7. Solve.

- $x^2 + 4x - 5 \leq 0$
- $-2x^3 + x^2 + 13x + 6 < 0$
- $2x^3 + x^2 - 2x - 1 > 0$
- $x^3 - 5x + 4 \geq 0$

For help with questions 8 and 9, refer to Example 3.

8. Cookies are packaged in boxes that measure 18 cm by 20 cm by 6 cm.

A larger box is being designed by increasing the length, width, and height of the smaller box by the same length so that the volume is at least  $5280 \text{ cm}^3$ . What are the minimum dimensions of the larger box?



9. The price,  $p$ , in dollars, of a stock  $t$  years after 1999 can be modelled by the function  $p(t) = 0.5t^3 - 5.5t^2 + 14t$ . When will the price of the stock be more than \$90?

## ✓ Achievement Check

10. a) Solve the inequality  $x^3 - 5x^2 + 2x + 8 < 0$  by  
i) using intervals  
ii) considering all cases  
b) How are the two methods the same? How are they different?

## C Extend and Challenge

11. a) How many cases must be considered when solving  $(x + 4)(x - 2)(x + 1)(x - 1) \leq 0$ ? Justify your answer.

- b) Would it be more efficient to solve this inequality using intervals? Justify your answer.

12. Solve  $x^5 + 7x^3 + 6x < 5x^4 + 7x^2 + 2$ .

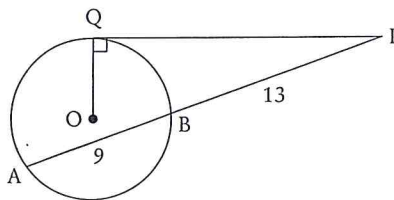
13. A demographer develops a model for the population,  $P$ , of a small town  $n$  years from today such that  $P(n) = -0.15n^5 + 3n^4 + 5560$ .

- When will the population of the town be between 10 242 and 25 325?
- When will the population of the town be more than 30 443?
- Will the model be valid after 20 years? Explain.

14. Write two possible quartic inequalities, one using the less than or equal to symbol ( $\leq$ ) and the other using the greater than or equal to symbol ( $\geq$ ), that correspond to the following solution:

$$-6 - \sqrt{2} < x < -6 + \sqrt{2} \text{ or } 6 - \sqrt{2} < x < 6 + \sqrt{2}$$

15. **Math Contest** Determine the exact length of PQ in the figure.



16. **Math Contest** Determine an equation for the line that is tangent to the circle with equation  $x^2 + y^2 - 25 = 0$  and passes through the point  $(4, -3)$ .