EX2.6 (p. 138 – 139) # 1 – 13

Case 2

Both factors are non-positive, and w is positive (because w represents the width).

$0 < w \leq 8$ $\qquad$ $w^2 + 19w + 176 \leq 0$

$w^2 + 19w + 176 \leq 0$ is not possible for any values of w. There is no solution.

So, the possible solution is $w \geq 8$.

When $w = 8$, $h = \frac{1}{4}(8) + 2 = 4$ and $l = 4(2) + 12 = 20$.

The dimensions of the excavation that give a volume of at least 1408 cm^3 are width 8 m, depth 4 m, and length 20 m.

KEY CONCEPTS

- Factorable inequalities can be solved algebraically by
  - considering all cases
  - using intervals and then testing values in each interval
- Tables and number lines can help organize intervals to provide a visual clue to solutions.

Communicate Your Understanding

C1 Why is it necessary to reverse an inequality sign when each side is multiplied or divided by a negative value? Support your answer with examples.

C2 What are the similarities between solving a linear inequality and solving a polynomial inequality?

C3 Which method is more efficient for solving factorable inequalities algebraically, using cases or using intervals? Explain.

A Practise

For help with question 1, refer to Example 1.

1. Solve each inequality. Show each solution on a number line.

a) $x + 3 \leq 5$ $\qquad$ **b)** $2x + 1 > -4$

c) $5 - 3x \geq 6$ $\qquad$ **d)** $7x < 4 + 3x$

e) $2 - 4x > 5x + 20$ $\qquad$ **f)** $2(1 - x) \leq x - 8$

For help with questions 2 to 4, refer to Example 2.

2. Solve by considering all cases. Show each solution on a number line.

a) $(x + 2)(x - 4) > 0$

b) $(2x + 3)(4 - x) \leq 0$

3. Solve using intervals. Show each solution on a number line.

a) $(x + 3)(x - 2) > 0$

b) $(x - 6)(x - 9) \leq 0$

c) $(4x + 1)(2 - x) \geq 0$

4. Solve.

a) $(x + 2)(3 - x)(x + 1) < 0$

b) $(-x + 1)(3x - 1)(x + 7) \geq 0$

c) $(7x + 2)(1 - x)(2x + 5) > 0$

d) $(x + 4)(-3x + 1)(x + 2) \leq 0$

B) Connect and Apply

5. Solve by considering all cases. Show each solution on a number line.

a) $x^2 - 8x + 15 \geq 0$

b) $x^2 - 2x - 15 < 0$

c) $15x^2 - 14x - 8 \leq 0$

d) $x^3 - 2x^2 - 5x + 6 < 0$

e) $2x^3 + 3x^2 - 2x - 3 \geq 0$

6. Solve using intervals.

a) $x^3 + 6x^2 + 7x + 12 \geq 0$

b) $x^3 + 9x^2 + 26x + 24 < 0$

c) $5x^3 - 12x^2 - 11x + 6 \leq 0$

d) $6x^4 - 7x^3 - 4x^2 + 8x + 12 > 0$

7. Solve.

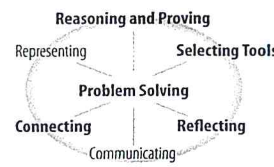
a) $x^2 + 4x - 5 \leq 0$

b) $-2x^3 + x^2 + 13x + 6 < 0$

c) $2x^3 + x^2 - 2x - 1 > 0$

d) $x^3 - 5x + 4 \geq 0$

For help with questions 8 and 9, refer to Example 3.



8. Cookies are packaged in boxes that measure 18 cm by 20 cm by 6 cm. A larger box is being designed by increasing the length, width, and height of the smaller box by the same length so that the volume is at least 5280 cm^3. What are the minimum dimensions of the larger box?

9. The price, p, in dollars, of a stock t years after 1999 can be modelled by the function $p(t) = 0.5t^3 - 5.5t^2 + 14t$. When will the price of the stock be more than \$90?

✓ Achievement Check

10. a) Solve the inequality $x^3 - 5x^2 + 2x + 8 < 0$ by

i) using intervals

ii) considering all cases

b) How are the two methods the same? How are they different?

C) Extend and Challenge

11. a) How many cases must be considered when solving $(x + 4)(x - 2)(x + 1)(x - 1) \leq 0$? Justify your answer.

b) Would it be more efficient to solve this inequality using intervals? Justify your answer.

12. Solve $x^5 + 7x^3 + 6x < 5x^4 + 7x^2 + 2$.

13. A demographer develops a model for the population, P, of a small town n years from today such that $P(n) = -0.15n^5 + 3n^4 + 5560$.

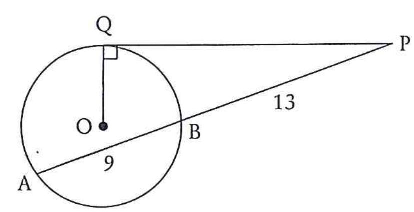
a) When will the population of the town be between 10 242 and 25 325?

b) When will the population of the town be more than 30 443?

c) Will the model be valid after 20 years? Explain.

14. Write two possible quartic inequalities, one using the less than or equal to symbol ($\leq$) and the other using the greater than or equal to symbol ($\geq$), that correspond to the following solution:
$-6 - \sqrt{2} < x < -6 + \sqrt{2}$ or
$6 - \sqrt{2} < x < 6 + \sqrt{2}$

15. Math Contest Determine the exact length of PQ in the figure.



16. Math Contest Determine an equation for the line that is tangent to the circle with equation $x^2 + y^2 - 25 = 0$ and passes through the point $(4, -3)$.