EX3.4 (p. 183–185) #1–15

KEY CONCEPTS

- To solve rational equations algebraically, start by factoring the expressions in the numerator and denominator to find asymptotes and restrictions.
- Next, multiply both sides by the factored denominators, and simplify to obtain a polynomial equation. Then, solve using techniques from Chapter 2.
- For rational inequalities:
  - It can often help to rewrite with the right side equal to 0. Then, use test points to determine the sign of the expression in each interval.
  - If there is a restriction on the variable, you may have to consider more than one case. For example, if $\frac{a}{x-k} < b$, case 1 is $x > k$ and case 2 is $x < k$.
- Rational equations and inequalities can be solved by studying the key features of a graph with paper and pencil or with the use of technology.
- Tables and number lines can help organize intervals and provide a visual clue to solutions.

Communicate Your Understanding

C1 Describe the process you would use to solve $\frac{2}{x-1} = \frac{3}{x+5}$.

C2 Explain why $\frac{1}{x^2 + 2x + 9} < 0$ has no solution.

C3 Explain the difference between the solution to the equation $\frac{4}{x-5} = \frac{3}{x+4}$ and the solution to the inequality $\frac{4}{x-5} < \frac{3}{x+4}$.

A Practise

For help with questions 1 and 2, refer to Example 1.

1. Determine the x-intercept(s) for each function. Verify using technology.

a) $y = \frac{x+1}{x}$

b) $y = \frac{x^2 + x - 12}{x^2 - 3x + 5}$

c) $y = \frac{2x-3}{5x+1}$

d) $y = \frac{x}{x^2 - 3x + 2}$

2. Solve algebraically. Check each solution.

a) $\frac{4}{x-2} = 3$

b) $\frac{1}{x^2 - 2x - 7} = 1$

c) $\frac{2}{x-1} = \frac{5}{x+3}$

d) $x - \frac{5}{x} = 4$

e) $\frac{1}{x} = \frac{x-34}{2x^2}$

f) $\frac{x-3}{x-4} = \frac{x+2}{x+6}$

For help with question 3, refer to Example 2.

3. **Use Technology** Solve each equation using technology. Express your answers to two decimal places.

   a) $\frac{5x}{x-4} = \frac{3x}{2x+7}$

   b) $\frac{2x+3}{x-6} = \frac{5x-1}{4x+7}$

   c) $\frac{x}{x-2} = \frac{x^2-4x+1}{x-3}$

   d) $\frac{x^2-1}{2x^2-3} = \frac{2x^2+3}{x^2+1}$

For help with question 4, refer to Example 3.

4. Solve each inequality without using technology. Illustrate the solution on a number line.

   a) $\frac{4}{x-3} < 1$ b) $\frac{7}{x+1} > 7$

   c) $\frac{5}{x+4} \leq \frac{2}{x+1}$ d) $\frac{(x-2)(x+1)^2}{(x-4)(x+5)} \geq 0$

   e) $\frac{x^2-16}{x^2-4x-5} > 0$ f) $\frac{x-2}{x} < \frac{x-4}{x-6}$

For help with question 5, refer to Example 4.

5. Solve each inequality using an interval table. Check using technology.

   a) $\frac{x^2+9x+14}{x^2-6+5} > 0$

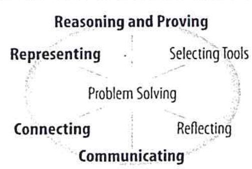
   b) $\frac{2x^2+5x-3}{x^2+8x+16} < 0$

   c) $\frac{x^2-3x-4}{x^2+11x+30} \leq 0$

   d) $\frac{3x^2-8x+4}{2x^2-9x-5} \geq 0$

B Connect and Apply

6. Write a rational equation that cannot have $x = 3$ or $x = -5$ as a solution. Explain your reasoning.



7. Solve $\frac{x}{x+1} < \frac{2x}{x-2}$ by graphing the functions $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{2x}{x-2}$ with or without using technology. Determine the points of intersection and when $f(x) < g(x)$.

8. Use the method from question 7 to solve $\frac{x}{x-3} > \frac{3x}{x+5}$.

9. Solve and check.

   a) $\frac{1}{x} + 3 = \frac{2}{x}$ b) $\frac{2}{x+1} + 5 = \frac{1}{x}$

   c) $\frac{12}{x} + x = 8$ d) $\frac{x}{x-1} = 1 - \frac{1}{1-x}$

   e) $\frac{2x}{2x+3} - \frac{2x}{2x-3} = 1$

   f) $\frac{7}{x-2} - \frac{4}{x-1} + \frac{3}{x+1} = 0$

10. Solve. Illustrate graphically.

    a) $\frac{2}{x} + 3 > \frac{29}{x}$ b) $\frac{16}{x} - 5 < \frac{1}{x}$

    c) $\frac{5}{6x} + \frac{2}{3x} > \frac{3}{4}$ d) $6 + \frac{30}{x-1} < 7$

11. The ratio of $x + 2$ to $x - 5$ is greater than $\frac{3}{5}$. Solve for x.

12. Compare the solutions to $\frac{2x-1}{x+7} > \frac{x+1}{x+3}$ and $\frac{2x-1}{x+7} < \frac{x+1}{x+3}$.

13. Compare the solutions to $\frac{x+1}{x-4} \leq \frac{x-3}{x+5}$ and $\frac{x-4}{x+1} \leq \frac{x+5}{x-3}$.

14. A number x is the harmonic mean of two numbers a and b if $\frac{1}{x}$ is the mean of $\frac{1}{a}$ and $\frac{1}{b}$.

    a) Write an equation to represent the harmonic mean of a and b.

    b) Determine the harmonic mean of 12 and 15.

    c) The harmonic mean of 6 and another number is 1.2. Determine the other number.

15. Chapter Problem Light pollution is caused by many lights being on in a concentrated area. Think of the night sky in the city compared to the night sky in the country. Light pollution can be a problem in cities, as more and more bright lights are used in such things as advertising and office buildings. The intensity of illumination is inversely proportional to the square of the distance to the light source and is defined by the formula $I = \frac{k}{d^2}$, where I is the intensity, in lux; d is the distance from the source, in metres; and k is a constant. When the distance from a certain light source is 10 m, the intensity is 900 lux.

a) Determine the intensity when the distance is

i) 5 m **ii)** 200 m

b) What distance, or range of distance, results in an intensity of

i) 4.5 lux? **ii)** at least 4500 lux?

16. The relationship between the object distance, d, and image distance, I, both in centimetres, for a camera with focal length 2.0 cm is defined by the relation $d = \frac{2.0I}{I - 2.0}$. For what values of I is d greater than 10.0 cm?

✓ Achievement Check

17. Consider the functions $f(x) = \frac{1}{x} + 4$ and $g(x) = \frac{2}{x}$. Graph f and g on the same grid.

a) Determine the points of intersection of the two functions.

b) Show where $f(x) < g(x)$.

c) Solve the equation $\frac{1}{x} + 4 = \frac{2}{x}$ to check your answer to part a).

d) Solve the inequality $\frac{1}{x} + 4 < \frac{2}{x}$ to check your answer to part b).

C Extend and Challenge

18. a) A rectangle has perimeter 64 cm and area 23 cm². Solve the following system of equations to find the rectangle's dimensions.

$$l = \frac{23}{w}$$

$$l + w = 32$$

b) Solve the system of equations.

$$x^2 + y^2 = 1$$

$$xy = 0.5$$

19. Use your knowledge of exponents to solve.

a) $\frac{1}{2^x} = \frac{1}{x + 2}$

b) $\frac{1}{2^x} > \frac{1}{x^2}$

20. Determine the region(s) of the Cartesian plane for which

a) $y > \frac{1}{x^2}$

b) $y \le x^2 + 4$ and $y \ge \frac{1}{x^2 + 4}$

21. Math Contest In some situations, it is convenient to express a rational expression as the sum of two or more simpler rational expressions. These simpler expressions are called *partial fractions*. Partial fractions are used when, given $\frac{P(x)}{D(x)}$, the degree of $P(x)$ is less than the degree of $D(x)$.

Decompose each of the following into partial fractions. Check your solutions by graphing the equivalent function on a graphing calculator. Hint: Start by factoring the denominator, if necessary.

a) $f(x) = \frac{5x + 7}{x^2 + 2x - 3}$

b) $g(x) = \frac{7x + 6}{x^2 - x - 6}$

c) $h(x) = \frac{6x^2 - 14x - 27}{(x + 2)(x - 3)^2}$