

Example 5 Solve for an Unknown Coefficient

Determine the value of k such that when $3x^4 + kx^3 - 7x - 10$ is divided by $x - 2$, the remainder is 8.

Solution

Let $P(x) = 3x^4 + kx^3 - 7x - 10$.

By the remainder theorem, when $P(x)$ is divided by $x - 2$, the remainder is $P(2)$.

Solve $P(2) = 8$.

$$\begin{aligned} 3(2)^4 + k(2)^3 - 7(2) - 10 &= 8 \\ 48 + 8k - 14 - 10 &= 8 \\ 24 + 8k &= 8 \\ 8k &= -16 \\ k &= -2 \end{aligned}$$

The value of k is -2 .

KEY CONCEPTS

- Long division can be used to divide a polynomial by a binomial.
- The result of the division of a polynomial function $P(x)$ by a binomial of the form $x - b$ can be written as $P(x) = (x - b)Q(x) + R$ or $\frac{P(x)}{x - b} = Q(x) + \frac{R}{x - b}$, where $Q(x)$ is the quotient and R is the remainder.
- To check the result of a division, use $\text{divisor} \times \text{quotient} + \text{remainder} = \text{dividend}$.
- The remainder theorem states that when a polynomial function $P(x)$ is divided by $x - b$, the remainder is $P(b)$, and when it is divided by $ax - b$, the remainder is $P\left(\frac{b}{a}\right)$, where a and b are integers and $a \neq 0$.

Communicate Your Understanding

- Explain why there is a restriction on the divisor of a polynomial function. How is the restriction determined?
- When and why might it be necessary to use a placeholder when dividing a polynomial by a binomial?
- Describe the error in this statement:

$$\frac{x^3 + 3x^2 - 2x - 1}{x - 2} = (x^2 + 5x + 8) + 5$$

- C4** Given a polynomial function $P(x)$ such that $P(-3) = 0$, what are the divisor and the remainder? What is the relationship between the divisor and $P(x)$?
- C5** Identify the dividend, divisor, quotient, and remainder in each statement.
- $\frac{6x^2 + 5x - 7}{3x + 1} = 2x + 1 - \frac{8}{3x + 1}$
 - $12x^3 + 2x^2 + 11x + 14 = (3x + 2)(4x^2 - 2x + 5) + 4$
 - $\frac{5x^3 - 7x^2 - x + 6}{x - 1} = 5x^2 - 2x - 3 + \frac{3}{x - 1}$

A Practise

For help with questions 1 and 2, refer to Example 1.

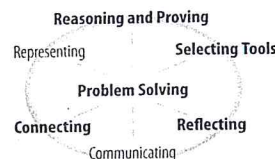
- Divide $x^3 + 3x^2 - 2x + 5$ by $x + 1$. Express the result in quotient form.
 - Identify any restrictions on the variable.
 - Write the corresponding statement that can be used to check the division.
 - Verify your answer.
- Divide $3x^4 - 4x^3 - 6x^2 + 17x - 8$ by $3x - 4$. Express the result in quotient form.
 - Identify any restrictions on the variable.
 - Write the corresponding statement that can be used to check the division.
 - Verify your answer.

For help with question 3, refer to Example 2.

- Perform each division. Express the result in quotient form. Identify any restrictions on the variable.
 - $x^3 + 7x^2 - 3x + 4$ divided by $x + 2$
 - $6x^3 + x^2 - 14x - 6$ divided by $3x + 2$
 - $10x^3 + 11 - 9x^2 - 8x$ divided by $5x - 2$
 - $11x - 4x^4 - 7$ divided by $x - 3$
 - $3 + x^2 + 7x + 6x^3$ divided by $3x + 2$
 - $8x^3 + 4x^2 - 31$ divided by $2x - 3$
 - $6x^2 - 6 + 8x^3$ divided by $4x - 3$
- Determine the remainder R so that each statement is true.
 - $(2x - 3)(3x + 4) + R = 6x^2 - x + 15$
 - $(x + 2)(x^2 - 3x + 4) + R = x^3 - x^2 - 2x - 1$
 - $(x - 4)(2x^2 + 3x - 1) + R = 2x^3 - 5x^2 - 13x + 2$

For help with questions 5 and 6, refer to Example 3.

- The volume, in cubic centimetres, of a rectangular box can be modelled by the polynomial expression $2x^3 + 17x^2 + 38x + 15$. Determine possible dimensions of the box if the height, in centimetres, is given by $x + 5$.
- The volume, in cubic centimetres, of a square-based box is given by $9x^3 + 24x^2 - 44x + 16$. Determine possible dimensions of the box if the area of the base, in square centimetres, is $9x^2 - 12x + 4$.



For help with questions 7 to 9, refer to Example 4.

- Use the remainder theorem to determine the remainder when $2x^3 + 7x^2 - 8x + 3$ is divided by each binomial. Verify your answer using long division.
 - $x + 1$
 - $x - 2$
 - $x + 3$
 - $x - 4$
 - $x - 1$
- Determine the remainder when each polynomial is divided by $x + 2$.
 - $x^3 + 3x^2 - 5x + 2$
 - $2x^3 - x^2 - 3x + 1$
 - $x^4 + x^3 - 5x^2 + 2x - 7$
- Use the remainder theorem to determine the remainder for each division.
 - $x^3 + 2x^2 - 3x + 9$ divided by $x + 3$
 - $2x^3 + 7x^2 - x + 1$ divided by $x + 2$
 - $x^3 + 2x^2 - 3x + 5$ divided by $x - 3$
 - $x^4 - 3x^2 - 5x + 2$ divided by $x - 2$

B Connect and Apply

For help with questions 10 to 12, refer to Example 5.

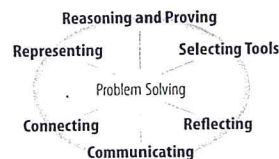
10. a) Determine the value of k such that when $P(x) = kx^3 + 5x^2 - 2x + 3$ is divided by $x + 1$, the remainder is 7.
b) Determine the remainder when $P(x)$ is divided by $x - 3$.
11. a) Determine the value of c such that when $f(x) = x^4 - cx^3 + 7x - 6$ is divided by $x - 2$, the remainder is -8 .
b) Determine the remainder when $f(x)$ is divided by $x + 2$.
c) **Use Technology** Verify your answer in part b) using a CAS.
12. For what value of b will the polynomial $P(x) = -2x^3 + bx^2 - 5x + 2$ have the same remainder when it is divided by $x - 2$ and by $x + 1$?
13. For what value of k will the polynomial $f(x) = x^3 + 6x^2 + kx - 4$ have the same remainder when it is divided by $x - 1$ and by $x + 2$?
14. a) Use the remainder theorem to determine the remainder when $2x^3 + 5x^2 - 6x + 4$ is divided by $2x + 1$.
b) Verify your answer in part a) using long division.
c) **Use Technology** Verify your answer in part a) using technology.
15. a) Use the remainder theorem to determine the remainder when $10x^4 - 11x^3 - 8x^2 + 7x + 9$ is divided by $2x - 3$.
b) **Use Technology** Verify your answer in part a) using long division or using a CAS.
16. a) Determine the remainder when $6x^3 + 23x^2 - 6x - 8$ is divided by $3x - 2$.
b) What information does the remainder provide about $3x - 2$? Explain.
c) Express $6x^3 + 23x^2 - 6x - 8$ in factored form.

17. **Chapter Problem** The packaging design team at Best of U has determined that a cost-efficient way of manufacturing cylindrical containers for their products is to have the volume, V , in cubic centimetres, modelled by $V(x) = 9\pi x^3 + 51\pi x^2 + 88\pi x + 48\pi$, where x is an integer such that $2 \leq x \leq 8$. The height, h , in centimetres, of each cylinder is a linear function given by $h(x) = x + 3$.
a) Determine the quotient $\frac{V(x)}{h(x)}$. Interpret this result.
b) Use your answer in part a) to express the volume of a container in the form $\pi r^2 h$.
c) What are the possible dimensions and volumes of the containers for the given values of x ?

CONNECTIONS

The formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius of the circular base and h is the height.

18. Jessica Zelinka, a Canadian heptathlete, won a gold medal in javelin throw at the Pan American games in 2007. Suppose $h(t) = -5t^2 + 15t + 1$ represents the approximate height, in metres, of a javelin t seconds after it is thrown.
a) Write a statement that corresponds to the quotient $\frac{h(t)}{t - b}$, where b is a positive integer.
b) Show that the statement in part a) may be written as $Q(t) = \frac{h(t) - h(b)}{t - b}$.
c) What is the geometric interpretation of $\frac{h(t) - h(b)}{t - b}$? Support your answer with a diagram.
d) Use the result of part c) to explain the physical meaning of $Q(t)$ for this situation.
e) Determine the remainder when $h(t)$ is divided by $t - 3$. Interpret the remainder for this situation.



19. The shot-put is another event in a heptathlon. Suppose $h(t) = -5t^2 + 8.3t + 1.2$ represents the approximate height, h , in metres, of a shot-put t seconds after it is thrown.

- a) Determine the remainder when $h(t)$ is divided by $t - 1.5$.
b) Use the results of question 18 to interpret your answer in part a) for this situation.

C Extend and Challenge

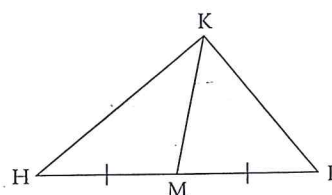
20. When the polynomial $mx^3 - 3x^2 + nx + 2$ is divided by $x + 3$, the remainder is -1 . When it is divided by $x - 2$, the remainder is -4 . Determine the values of m and n .
21. When the polynomial $3x^3 + ax^2 + bx - 9$ is divided by $x - 2$, the remainder is -5 . When it is divided by $x + 1$, the remainder is -16 . Determine the values of a and b .
22. When $3x^2 + 10x - 3$ is divided by $x + k$, the remainder is 5 . Determine the values of k .
23. Math Contest When a number, x , is divided by 4 , the remainder is 3 . Determine the remainder when $5x$ is divided by 4 .

24. Math Contest Determine the area, A , of a triangle with vertices $A(4, 6)$, $B(2, 3)$, and $C(8, 4)$ by applying Heron's formula,

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where a , b , and c are the side lengths and $s = \frac{1}{2}(a + b + c)$.

25. Math Contest In $\triangle HKL$, $\angle HKL = 90^\circ$. Prove that $HM = MK$.



CAREER CONNECTION

Since graduating from a 4-year environmental science program at the University of Ottawa, Chantal has been working to become a licensed environmental engineer. She works in water resources management and ensures that social, economic, environmental, and technical concerns are taken into account when water resources, such as reservoirs, are built and maintained. Chantal creates mathematical models of the water resource she is studying and tests them for various factors. For example, she may test the maximum storage capacity of a new reservoir or optimize the amount of water an existing reservoir should release.

