

EX 1.5 (p. 62 - 64) # 1 - 8, AND # 10

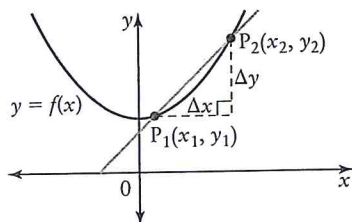
$$\begin{aligned}\text{Average rate of change} &= \frac{\Delta h}{\Delta t} \\ &= \frac{5.375 - 9.4}{2.5 - 2} \\ &= -8.05\end{aligned}$$

The average rate of change of the height of the football from 2 s to 2.5 s is -8.05 m/s.

- b) The average rate of change of the height of the football for $t \in [0, 0.5]$ corresponds to the slope of secant AB. The average rate of change is positive, as is the slope of AB. The height of the football is increasing. The average rate of change of the height of the football for $t \in [2, 2.5]$ corresponds to the slope of secant CD. The average rate of change is negative, as is the slope of the secant. The height of the football is decreasing.

KEY CONCEPTS

- A rate of change is a measure of how quickly one quantity (the dependent variable) changes with respect to another quantity (the independent variable).
- Average rates of change
 - represent the rate of change over a specified interval
 - correspond to the slope of a secant between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on a curve



$$\begin{aligned}\text{Average rate of change} &= \frac{\Delta y}{\Delta x} \\ &= \frac{y_2 - y_1}{x_2 - x_1}\end{aligned}$$

- An average rate of change can be determined by calculating the slope between two points given in a table of values or by using an equation.

Communicate Your Understanding

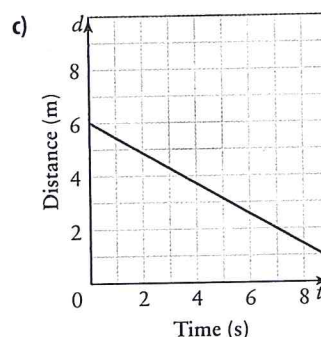
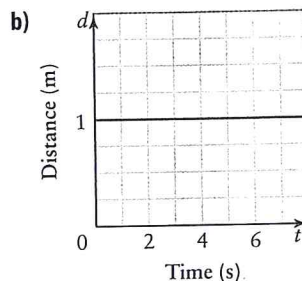
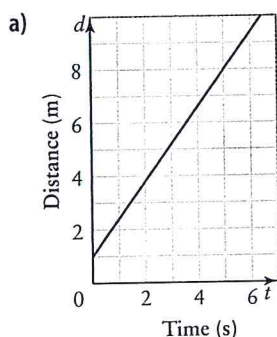
- Describe a situation for which the average rate of change is
 - constant and positive
 - constant and negative
 - zero
- State the average rate of change for this situation. When the change in the independent variable is -3 , the change in the dependent variable is 12 .
- What information is provided by the sign (positive or negative) of an average rate of change?
 - How can you tell from a graph if the average rate of change over an interval is positive or negative? Is it always possible to do so? Explain.

A Practise

- Which of the following does not represent a situation that involves an average rate of change? Justify your answer.
 - A child grows 8 cm in 6 months.
 - The temperature at a 750-m-high ski hill is 2°C at the base and -8°C at the top.
 - A speedometer shows that a vehicle is travelling at 90 km/h.
 - A jogger ran 23 km in 2 h.
 - The laptop cost \$750.
 - A plane travelled 650 km in 3 h.

For help with questions 2 and 3, refer to Example 1.

- Identify if the average rate of change for pairs of points along each graph is constant and positive, constant and negative, zero, or non-constant. Justify your response.



- Determine the average rate of change for two points on each line segment in question 2.
- In 1990, 16.2% of households had a home computer, while 66.8% of households had a home computer in 2003. Determine the average rate of change of the percent of households that had a home computer over this time period.

Source: Statistics Canada, Canada at a Glance 2006, page 9, Household facilities.

B Connect and Apply

For help with question 5, refer to Example 2.

- The table shows the percent of Canadian households that used e-mail from 1999 to 2003.

Year	Households (%)
1999	26.3
2000	37.4
2001	46.1
2002	48.9
2003	52.1

Source: Statistics Canada, Canada at a Glance, 2006, page 9, Household Internet use at home by Internet activity.

- Determine the average rate of change of the percent of households using e-mail from 1999 to 2003. What are the units for this average rate of change?
- Why might someone want to know the average rate of change found in part a)?
- Determine the average rate of change of the percent of households using e-mail for each pair of consecutive years from 1999 to 2003.
- Compare the values found in part c). Which value is the greatest? the least? What is the significance of these values?
- Compare the values found in part a) with those in part c). Explain any similarities or differences.

B Connect and Apply

For help with questions 6 to 8, refer to Example 3.

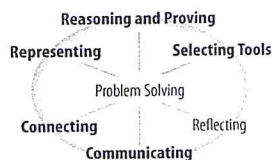
6. **Use Technology** The purchase price, P , of one share in a company at any time, t , in years, can be modelled by the function
- $$P(t) = -0.2t^3 + 2t^2 + 8t + 2, \quad t \in [0, 13].$$

- Graph the function.
- Use the graph to describe when the rate of change is positive, when it is zero, and when it is negative.
- Determine the average rate of change of the purchase price from
 - year 0 to year 5
 - year 5 to year 8
 - year 8 to year 10
 - year 8 to year 13
- When was the best time to buy shares? sell shares? Justify your answers.

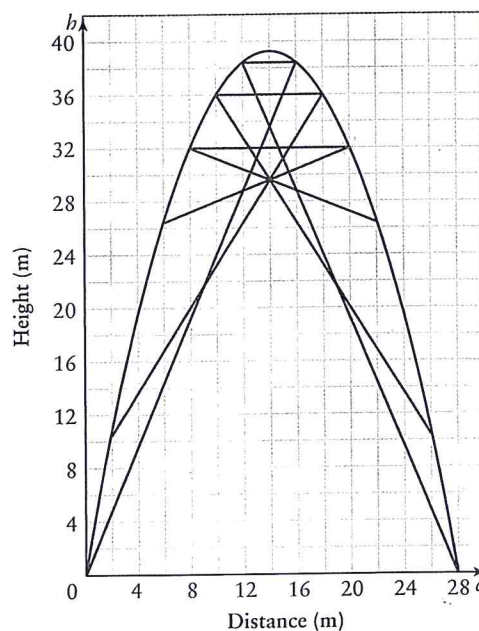
7. As a large snowball melts, its size changes. The volume, V , in cubic centimetres, is given by the equation

$V = \frac{4}{3}\pi r^3$, where r is the radius, in centimetres, and $r \in [0, 30]$. The surface area, S , in square centimetres, is given by the equation $S = 4\pi r^2$.

- What type of polynomial function does each formula represent? Sketch a graph of each function. State the domain and range.
- Determine the average rate of change of the surface area and of the volume as the radius decreases from
 - 30 cm to 25 cm
 - 25 cm to 20 cm
 Compare the change in surface area to the change in volume. Describe any similarities and differences.
- Determine the average rate of change of the surface area when the surface area decreases from 2827.43 cm^2 to 1256.64 cm^2 .
- Determine the average rate of change of the volume when the volume decreases from 1675.52 cm^3 to 942.48 cm^3 .
- Interpret your answers in parts c) and d).



8. A cyclist riding a bike at a constant speed along a flat road decelerates to climb a hill. At the top of the hill the cyclist accelerates and returns to the constant cruising speed along another flat road, and then accelerates down a hill. Finally, the cyclist comes to another hill and coasts to a stop. Sketch a graph of the cyclist's speed versus time and a graph of distance travelled versus time. Justify your sketches.
9. **Chapter Problem** A structural engineer designs a bridge to be built over a river. The following design, consisting of a parabola and crossbeams, represents the bridge's metal support structure.



- Determine an equation for the parabola in the design.
- What type of line does each crossbeam represent?
- Determine the slope of each crossbeam in the design. Describe your method.
- What do the slopes of the crossbeams represent?
- How is the symmetry of the parabola shown by the slopes of the crossbeams?