Section 1.4
Transformation

In this section we will be answering the following question: What are the roles of a, k, d, and c in polynomial functions of the form

$$\boldsymbol{y = a\,[k(x-d)]^n + c}$$

Values of c and d in $\boldsymbol{y = a\,[k(x-d)]^n + c}$

Example 1: Draw the following graphs: $y = x^3, y = x^3 + 2, y = (x+2)^3, y = (x+2)^3 + 2$

Value of c in $\boldsymbol{y = a\,[k(x-d)]^n + c}$	**Effect on the graph of** $\boldsymbol{y = x^n}$
$c < 0$	Graphs shifts to the top.
$c > 0$	Graphs shifts to the bottom.

Value of c in $\boldsymbol{y = a\,[k(x-d)]^n + c}$	**Effect on the graph of** $\boldsymbol{y = x^n}$
$d < 0$	Graphs shifts to the left.
$d > 0$	Graphs shifts to the right.

Values of a and k in $\boldsymbol{y = a\,[k(x-d)]^n + c}$

Example 2: Draw the following graphs: $\boldsymbol{y = x^3, y = (3x)^3, y = 3x^3, y = (\frac{1}{3}x)^3, y = \frac{1}{3}x^3}$

Value of a in $\boldsymbol{y = a\,[k(x-d)]^n + c}$	**Effect on the graph of** $\boldsymbol{y = x^n}$
$a > 1$	Vertically stretch the graph by factor **a**
$0 < a < 1$	Vertically compress the graph by factor a

Value of a in $\boldsymbol{y = a\,[k(x-d)]^n + c}$	**Effect on the graph of** $\boldsymbol{y = x^n}$
$k > 1$	Horizontally compress the graph by factor $\frac{1}{k}$
$0 < k < 1$	Horizontally stretch the graph by a factor of k

Reflection	**Effect on the graph of $y = x^n$**
$-h(x)$	Reflection in the x-axis.
$h(-x)$	Reflection in the y-axis.

Note: When n is even the graphs of polynomial functions of the form $\boldsymbol{y = a\,[k(x-d)]^n + c}$ are even functions and have a vertex at (d,c). The axis of symmetry is x=d.

For $a > 0$, the graph opens upward. The vertex is the minimum point on the graph and c is the minimum value. For $a < 0$, the graph opens downward. The vertex is the maximum point on the graph and c is the maximum value.

Example 1:

(a)Describe the transformations that must be applied to the graph of each function and write the corresponding equation.

(b) State the domain and range. State the vertex and the equation for the axis of symmetry for functions that are even.

(i) $f(x) = x^4, y = 2f\left[\frac{1}{3}(x-5)\right]$

(ii) $f(x) = x^5, y = \frac{1}{4}f[-2x+6] + 4$

a)

a)

b)

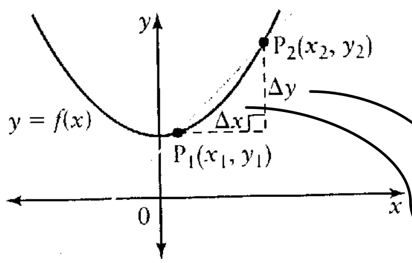
b)

Section 1.5
Slopes of Secants and Average Rate of Change

Rate of change: is a measure of the change in the dependant variable with respect to the change in the independent variable.

Secant: a line that connects two points on a curve.

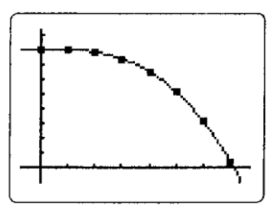
Figure 1:



$$\textbf{\textit{Average rate of change}} = \frac{\Delta y}{\Delta x} = \frac{\textit{change in } y}{\textit{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: A new antibacterial spray is tested on a bacterial culture. The table shows the population, P, of the bacterial culture t minutes after the spray is applied.

t (min)	P
0	800
1	799
2	782
3	737
4	652
5	515
6	314
7	37



a) Determine the average rate of change of the number of bacteria over the entire time period shown in table. Interpret the value for this situation.

b) Compare the average rate of change of the number of bacteria in the first 3 min and in the last 3 min.

c) How can you tell that this situation involves a non-constant rate of change by examining: (i) the table of values (ii) the graph (iii) the average rate of change

Section 1.6
Slopes of Tangents and Instantaneous Rate of Change

Example 1: Examine the following equation $h(t) = -4.9t^2 + 25t$ where h is height and t is seconds What is the instantaneous rate of changes at t=1 second.

This problem can be solved by simply using the slope formula : $slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

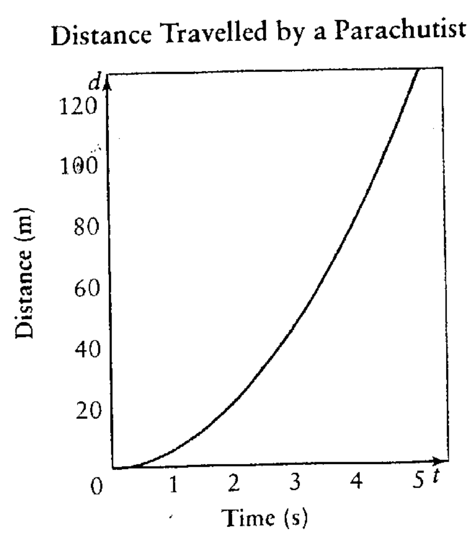
Complete the following table below.

Interval	Δh	Δt	Average rate of change, $\frac{\Delta h}{\Delta t}$
$1 \leq t \leq 2$			
$1 \leq t \leq 1.5$			
$1 \leq t \leq 1.1$			
$1 \leq t \leq 1.01$			
$1 \leq t \leq 1.001$			

How are the intervals in the first column changing? Draw the graph and sketch the secants. Sketch a tangent line that touches the graph at t=1 second.

Main Idea: As a point Q becomes very close to a tangent point P, the slope of the secant line becomes closer to the slope of the tangent line. Thus, the average rate of change between P and Q becomes closer to the value of the instantaneous rate of change at P.

Example 2: Examine the graph to the right which shows the approximate distance travelled by a parachutist in the first 5 seconds after jumping out of a helicopter. How fast was the parachutist travelling 2 sec after jumping out of the helicopter?



Solution: